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Transition Mechanisms in Conventional Hypersonic Wind Tunnels

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A recently developed theory that addresses bypass transition is used to study mechanisms responsible for transition in two-dimensional/axisymmetric flows in conventional hypersonic wind tunnels. It is shown that transition in such facilities, where the intensity is in excess of 1%, is a result of a combined bypass/second-mode mechanism. This mechanism is validated by comparing predictions of the theory with heat transfer measurements carried out for straight and flared sharp cones at zero angle of attack, a Mach number of 7.93, and unit Reynolds numbers of 1.6–8.2 $\times 10^6$/m. In general, good agreement with experiment is indicated.

Nomenclature

a	=	model constants (first-mode)
a_1–a_3	=	model constants defined in Eq. (6)
b	=	model constants (second-mode)
c_μ	=	constant, 0.09
k	=	turbulent kinetic energy, m^2/s^2
M	=	Mach number
$\tilde{m}$	=	rms of mass flux, kg/m$^2 \cdot$ s
$\bar{m}$	=	mean mass flux, kg/m$^2 \cdot$ s
P	=	pressure, Pa
q_w	=	heat flux at the wall, W/m^2
Re	=	Reynolds number
s_{ij}	=	strain tensor, s^{-1}
T	=	temperature, K
Tu	=	turbulent intensity
U_e	=	edge velocity, m/s
U_p	=	phase velocity, m/s
$\tilde{\boldsymbol{u}}_i$	=	mean velocity vector, m/s
x_t	=	location of transition onset, m
α	=	parameter defined in Eq. (6)
β	=	parameter defined in Eq. (9)
Γ	=	intermittency
δ	=	boundary-layer thickness, m
δ^*	=	displacement thickness, m
ζ	=	enstrophy, s^{-2}
λ	=	wavelength, m
μ	=	viscosity, kg/m $\cdot$ s
ν	=	kinematic viscosity, m^2/s
ξ	=	parameter defined in Eq. (9)
ρ	=	density, kg/m^3
τ	=	timescale, s
τ_k	=	decay time, s
ω	=	frequency, s^{-1}

Subscripts

e	=	boundary-layer edge
nt	=	nonturbulent
sm	=	second mode
t	=	turbulent

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Introduction

BY the use of results from linear stability theory, Warren and Hassan[1–3] developed an approach that is capable of economically calculating transitional flows using traditional Reynolds-averaged Navier–Stokes flowfield codes. In this approach, transitional flows are treated in a turbulencelike manner with the transitional eddy viscosity deduced from linear stability theory. A procedure capable of calculating transition onset and extent as part of the solution, without having to use stability codes or empirical correlations, resulted from applying this technique. Because of reliance on linear stability theory, applications of the approach are limited to situations where turbulence is a result of natural transition.

Recently, McDaniel and Hassan[4] extended the approach in Refs. 1–3 to bypass transition. The word bypass was used by Markovin[5] to describe transitional flows where the Tollmien–Schlicting (T–S) mechanism, which is the mechanism responsible for natural transition at low speeds in two-dimensional/axisymmetric flows, is completely bypassed. Such flows are characterized by high freestream turbulent intensities, greater than 1%, and are typical of flows in conventional wind tunnels, turbomachinery, and other flows subject to high-intensity disturbances. It is shown in Ref. 4 that bypass transition is a result of a receptivity mechanism, where the scale of the disturbance responsible for transition is quite different from that of a T–S wave. The model was calibrated and validated by comparing its prediction with test cases[6] that involve low-speed high freestream turbulent intensity flows in the presence and absence of pressure gradients.

It is generally believed that the growth of the second mode is the mechanism responsible for transition in two-dimensional/axisymmetric hypersonic flows irrespective of the turbulent intensity in the tunnel.[7] This observation is not in agreement with the work of Pate,[8] who demonstrated that freestream disturbance dominates the transition process in conventional wind tunnels. Further, the experiments of Lachowicz et al.[9] conducted at a Mach number of 6 do not support the observation made in Ref. 7. In the experiment of Ref. 9, the tunnel was run in quiet and conventional modes. Measurement of the adiabatic wall temperature on a 5-deg-half-angle flared cone clearly indicated that transition onset was dependent on the intensity of the tunnel. In addition, it has been shown in Ref. 10 that the second mode alone is not the only mode responsible for transition in quiet tunnels at $M = 6$. Thus, although the second mode may have greater growth rate in the linear region, it does not necessarily dominate the nonlinear interactions that result in transition.

The goal of the present work is to demonstrate that an approach based on a bypass/second mode is suited for studying two-dimensional/axisymmetric transitional flows in conventional hypersonic facilities. This objective was accomplished by comparing theoretical predictions with measurements performed by Kimmel[11] in the Arnold Engineering Development Center (AEDC) Tunnel B where the intensity lies in the range of 1–3.5% depending on the specific Mach and Reynolds number conditions.[12]

Formulation of the Problem

There were a number of considerations involved in developing the model of Ref. 4. The first consideration dealt with receptivity. It was determined that bypass transition was a result of entraining disturbances into the boundary layer thereby triggering perturbations that amplify and not a result of diffusion into the boundary layer. When it was determined that it was a receptivity mechanism, the next step in the development was to replace the T–S mechanism in the formulation of Refs. 1–3 by another strong amplifying mechanism.

To illustrate how the model of Ref. 4 was arrived at, a brief review of the Warren and Hassan model[1–3] is presented. In this model, the turbulent viscosity μ_t is replaced by

$$(1-\Gamma)\mu_{\text{nt}} + \Gamma\mu_t$$

where Γ is the intermittency or the fraction of the time the flow is turbulent. The eddy viscosity μ_{nt} is

$$\mu_{\text{nt}} = c_\mu \rho k \tau_{\text{nt}}, \qquad c_\mu = 0.09 \tag{1}$$

where k is the fluctuating kinetic energy per unit mass and τ_{nt} is the timescale of the nonturbulent fluctuations. For transition resulting from T–S waves,

$$\tau_{\text{nt}} = a/\omega, \qquad \omega\nu\big/U_e^2 = 0.48 Re_x^{-0.65} \tag{2}$$

where a is a model constant that depends on the freestream intensity, Re_x is the Reynolds number, and ω is the frequency of the most amplified mode. The correlation indicated in Eq. (2) was derived in Ref. 13 using results obtained by Mack.[14] Similarly, the dissipation timescale in the k equation was chosen as[1–3]

$$1/\tau_k = (1-\Gamma)(1/\tau_{k,\text{nt}}) + \Gamma(1/\tau_{k,t}) \tag{3}$$

where for T–S waves

$$\frac{1}{\tau_{k,\text{nt}}} = a\frac{\nu_{\text{nt}}}{\nu}S,\ S^2 = S_{ij}S_{ij},\ S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{4}$$

For bypass transition, Eqs. (2) and (4) were modified to

$$\tau_{\text{nt}} = a_1\left(\nu\big/U_e^2\right)Re_x^a, \qquad 1/\tau_{k,\text{nt}} = [a_2(\nu_{\text{nt}}/\nu) + a_3\Gamma]S \tag{5}$$

where a_1, a_2, a_3, and α are model constants.

These model constants were determined in Ref. 4 and have the values

$$a_1 = 0.0012, \qquad a_2 = 0.0036, \qquad a_3 = 0.34$$

$$\alpha = 0.9448\, Tu^{\frac{1}{8}} \tag{6}$$

where Tu is the turbulent intensity. No adjustments in these values were made in this investigation. As indicated earlier, these constants were arrived at from a consideratin of low-speed flows in the presence and absence of pressure gradients.[6] When the results obtained here are examined, it appears that these model constants are valid for a wide range of Reynolds and Mach numbers.

It is assumed that transition to turbulence in conventional hypersonic facilities is a result of the bypass mechanism described earlier and a second-mode mechanism. The contribution of the second mode to τ_{nt} is given by[10]

$$\tau_{\text{sm}} = b/\omega_{\text{sm}}, \qquad \omega_{\text{sm}} = U_p/\lambda \tag{7}$$

where U_p is about 0.94 times the edge velocity; λ is approximately 2δ, where δ is the boundary-layer thickness determined by the location where the change in vorticity relative to the vorticity is 10^{-4} or less; and b is a model constant equal to 0.054. This value was arrived at in Ref. 10 from consideration of flows in the NASA $M = 6$ quiet tunnel.[15] The reason for this choice is based on the consideration that only a small portion of the spectrum is in the second-mode frequency range (frequencies more than 80–100 kHz) (Ref. 7). Thus, for the hypersonic flow calculations presented here, τ_{nt} is chosen as

$$\tau_{\text{nt}} = a_1\left(\nu\big/U_e^2\right)Re_x^\alpha + 0.054/\omega_{\text{sm}} \tag{8}$$

Intermittency and Onset Prediction Criteria

The intermittency Γ employed here is a modification of that developed by Dhawan and Narasimha,[16] that is,

$$\Gamma(x) = 1 - \exp(-0.412\xi^2), \qquad \xi = \max(x - x_t, 0)/\beta \tag{9}$$

where β is the characteristic extent of the transitional region and is determined from the correlation

$$Re_\beta = 9.0 Re_{x_t}^{0.75} \tag{10}$$

with x_t being the location where turbulent spots first appear, or the onset of transition. In this work, x_t is determined as part of the solution.

Two modifications were made in the expression for β to address the influence of high intensity and high Mach numbers. Increased intensity has the tendency of shortening the transitional region whereas increased Mach numbers have the opposite effect. Based on correlations discussed by Malye,[17] the following adjustments were made in this work. If β_0 is the value of β that follows from Eq. (10), then

$$\beta = \begin{cases} \beta_0, & Tu \le 0.5 \\ \beta_0/2Tu, & Tu \ge 0.5 \end{cases} \tag{11}$$

The Mach number correction employed here is

$$\beta = \begin{cases} \beta_0, & M_e \le 7^{\frac{1}{2}} \\ \frac{5}{12}\beta_0\left\{1 + [(\gamma-1)/2]M_e^2\right\}, & M_e > 7^{\frac{1}{2}} \end{cases} \tag{12}$$

where M_e is the edge Mach number.

A minimum heat transfer criterion is used to determine transition onset. This criterion is appropriate when transition takes place on the straight part of the cone. Because heat flux increases in the presence of adverse pressure gradients at high Mach numbers, a minimum heat transfer criterion is inappropriate for such flows. An alternative criterion that sets

$$\nu_{\text{nt}}/\nu = 1.0 \tag{13}$$

at onset was developed in Ref. 4 to address flows where the pressure gradient is different from zero. This equation was developed for low-speed flow and, as such, does not hold for hypersonic flows. Thus, an onset criterion to determine transition onset when transition takes place on the flared portion of the cone is yet to be developed.

Results and Discussion

Intensity is not measured at hypersonic Mach numbers. Instead, the root mean square of mass flux $\tilde{m}$, over the mean flux $\bar{m}$, is measured. Relating $\tilde{m}/\bar{m}$ to intensity at hypersonic Mach numbers is not straight forward. This is because density fluctuations cannot be directly related to velocity fluctuations. For low speeds, this ratio reduces to the intensity. Thus, for the current purposes, $\tilde{m}/\bar{m}$ is a measure of the intensity.

As indicated in Ref. 12, the intensity depends on both Mach and Reynolds numbers. Although the intensity was not measured in Ref. 11, it was indicated by the author to be in the range 1–1.5%. A value of $Tu = 1.25$ is selected for all test cases considered here.

The numerical procedure used in Ref. 10 is employed here. The current model for bypass/second-mode transition mechanism was incorporated into Olynick and Hassan's two-dimensional/axisymmetric implicit solver for hypersonic flows.[18] It has been shown in Ref. 10 that a grid of the size 201×91 (along and normal to the surface) provides a grid-resolved solution. Further grid studies involving an increase of 25% in grid size in each direction were conducted to confirm grid convergence. All results presented employ a 201×91 grid.

Two 7-deg-half-angle sharp cones are considered in this investigation: a straight cone and a flared cone designated by $\mathrm{d}P/\mathrm{d}x = 4$ in Ref. 11. This designation corresponds to the nondimensional pressure gradient parameter $(L/C_{\text{pc}})\,\mathrm{d}C_p/\mathrm{d}x$, where L is the cone length and C_{pc} is the pressure coefficient on the conical forebody. Schematics of these cones are shown in Fig. 1 using coordinates

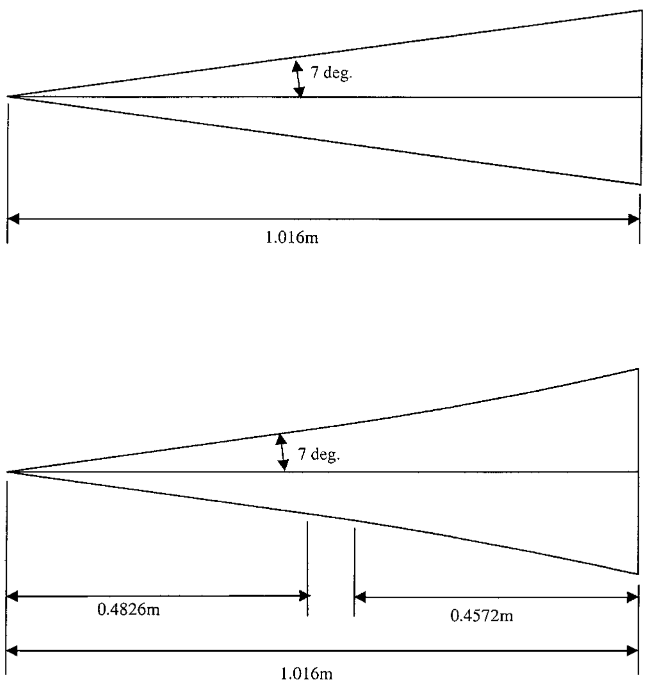

Fig. 1 Straight and flared cone configurations.

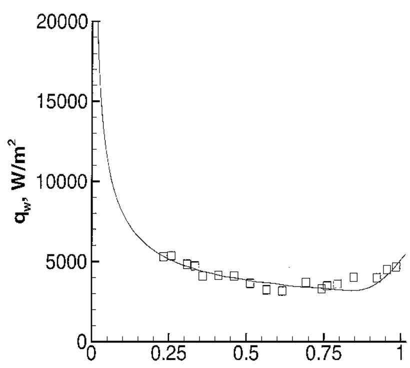

Fig. 2 Computed and measured heat flux, straight cone: *Re*/*L* = 3.3 × 10^6/m.

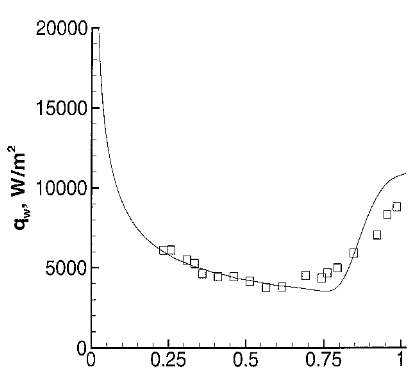

Fig. 3 Computed and measured heat flux, straight cone: *Re*/*L* = 3.9 × 10^6/m.

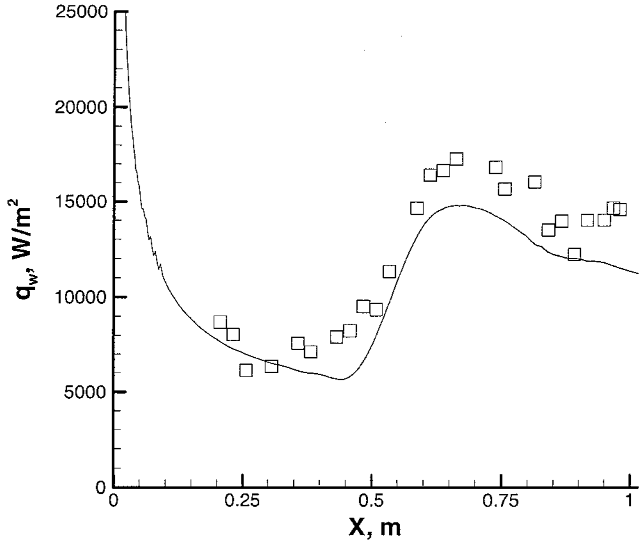

Fig. 4 Computed and measured heat flux, straight cone: *Re*/*L* = 6.6 × 10^6/m.

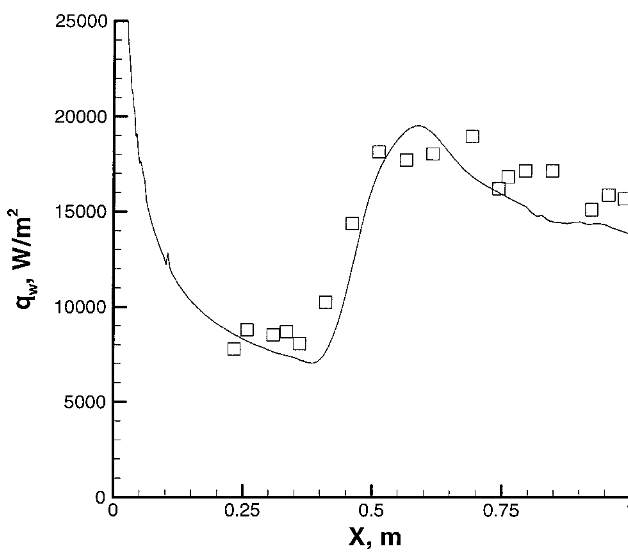

Fig. 5 Computed and measured heat flux, straight cone: *Re*/*L* = 8.2 × 10^6/m.

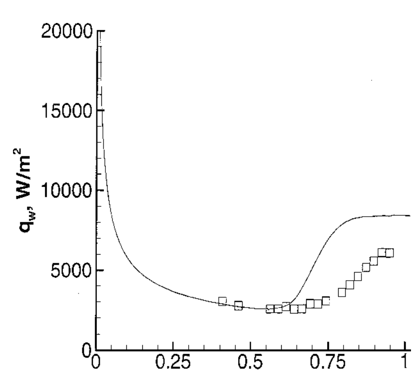

Fig. 6 Computed and measured heat flux, flared cone: *Re*/*L* = 1.6 × 10^6/m.

given in Ref. 11. Eight test cases were considered: four for the straight cone at unit Reynolds numbers $Re/L = 3.3$, 3.9, 6.6, and 8.2×10^6/m and four for the flared cone with $Re/L = 1.6$, 3.3, 4.9, and 6.6×10^6/m. For the eight cases considered, the freestream Mach number $M = 7.93$, stagnation temperature $T_0 = 728$ K, and the value of wall to stagnation temperature $T_w/T_0 = 0.42$.

Figures 2–5 compare wall heat flux q_w with experiment for the straight cone. The models were instrumented with Schmidt–Boelter heat transfer gauges, which also provided surface temperature measurements. The gauges were installed at 50.8-mm intervals from $x/L = 0.3$ to 0.45, and at 25.4-mm intervals between $x/L = 0.55$ and 0.925. According to Ref. 11, the accuracy of the heat transfer gauges is $\pm 10\%$. Moreover, minimum error in determining transition onset location is 50.8 mm, which corresponds to the spacings of the heat transfer gauges. As may be observed in Figs. 2–5, and considering the experimental uncertainties, good agreement is indicated for all Reynolds numbers considered. This level of agreement lends

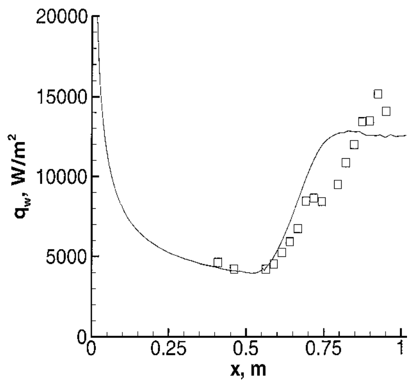

Fig. 7 Computed and measured heat flux, flared cone: *Re/L* = 3.3 × 10⁶/m.

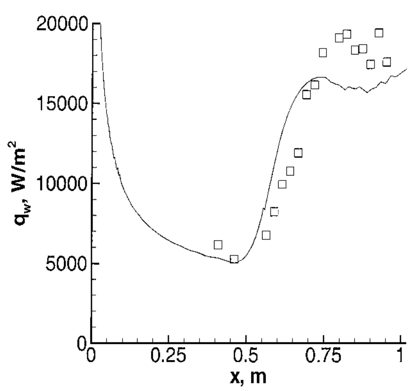

Fig. 8 Computed and measured heat flux, flared cone: *Re/L* = 4.9 × 10⁶/m.

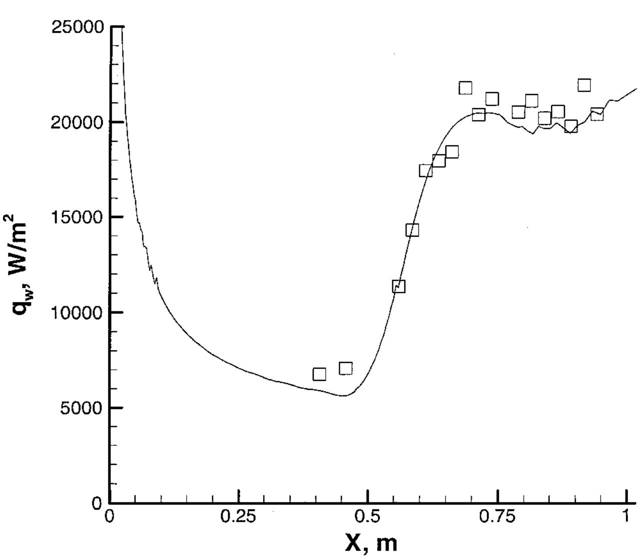

Fig. 9 Computed and measured heat flux flared cone: *Re/L* = 6.6 × 10⁶/m.

support to the accuracy of the proposed transitional mechanism and to the procedure used in determining onset and extent.

Figures 6–9 compare results for the flared cone cases. For the lower Reynolds numbers, transition takes place on the flare. For such cases, onset prediction is not correct because the criterion used to determine onset, that is, minimum heat flux, is not appropriate. Good agreement (within 5–10%) is indicated for the higher Reynolds numbers, where transition takes place on the forecone.

The present model can also be implemented for an assumed transition onset and is accomplished by setting the value of x_t in Eq. (9). Figure 10 shows a comparison between theory and experiment for the flared configuration at $Re/L = 1.6 \times 10^6$/m, where the transition is set at 0.7 m. This case corresponds to the lowest unit Reynolds number in the experiment. As is seen from Fig. 10, good agreement with experiment is indicated (within 5–10%). Thus, the inability to predict transition onset for this case is not a result of an inadequate model, rather, it is a result of an inadequate onset criterion.

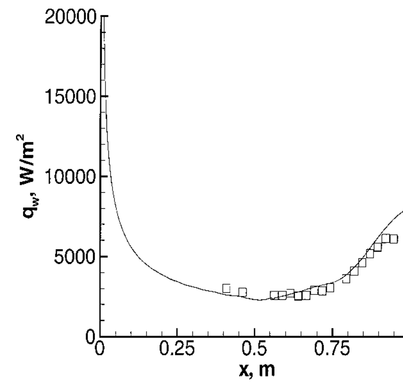

Fig. 10 Computed and measured heat flux (x_t = 0.7 m), flared cone: *Re/L* = 1.6 × 10⁶/m.

Conclusions

The present work suggests that a combination bypass/second-mode mechanism was responsible for transition in conventional hypersonic facilities. This conclusion is arrived at by examining flows past straight and flared cones at $M_\infty = 7.93$ for a range of Reynolds numbers and complements an earlier conclusion (reached in Ref. 10), that a combination first oblique mode/second mode is the mechanism responsible for transition in quiet wind tunnels. Finally, with this development, there is available to an analyst a conceptually simple and computationally efficient approach for the calculation of transitional flows that are a result of either natural or bypass transitions.

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